

Fig. 7. ($f_{\text{HFS}} - f_{\text{DS}}$) plotted vs atomic number at $\sin \theta/\lambda = 0.20$ and 0.70 \AA^{-1} .

effects are negligible and these two models differ only in this respect. The differences are essentially zero. (The small deviation from zero of the plotted differences at low atomic number arises for the most part from small inaccuracies in the analytic fits used to compute the differences). At the higher atomic numbers the difference is always negative because the relativistic DS atomic model is more compact than the non-relativistic HFS model.

As a general rule, the inclusion of exchange has a greater effect on the scattering factors than does relativity. At the heaviest elements the two effects appear to be comparable.

Conclusions

Crystallographers now are overwhelmed with scattering factors and one might well ask which of these he should use. For the lighter elements, up to Rb^+ , the values given in *International Tables for X-ray Crystallography* (1962) are the best ones, for they have been computed from Hartree-Fock wave functions and in some cases from even more accurate wave functions. For these lighter elements, relativistic effects are trivial.

For heavier elements it is suggested that the Dirac-Slater scattering factors are to be preferred because relativistic effects are accounted for and at least an approximation for exchange has been made.

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The Crystal Structure of [18]annulene. I. X-ray Study

BY JUDITH BREGMAN*, F. L. HIRSHFELD, D. RABINOVICH AND G. M. J. SCHMIDT

Department of X-ray Crystallography, Weizmann Institute of Science, Rehovoth, Israel

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The crystal structure of [18] annulene has been determined by least-squares analysis of photographic X-ray data recorded at the temperature of boiling nitrogen. The molecules occupy crystallographic centers of symmetry in a structure closely resembling that of coronene. This molecular symmetry rules out the possibility of a structure with alternate long and short C-C bonds. Final carbon coordinates have estimated standard deviations averaging less than 0.004 \AA ; the hydrogen atoms have not been reliably located.

Introduction

The synthesis by Sondheimer and his collaborators (Sondheimer & Wolovsky, 1959; Sondheimer & Gao-

* Present address: Department of Physics, Polytechnic Institute of Brooklyn, Brooklyn 1, N.Y., U.S.A.

ni, 1960, 1961, 1962; Sondheimer, Wolovsky & Gaoni, 1960; Sondheimer, Wolovsky & Amiel, 1962; Sondheimer, 1963) of the monocyclic alternant hydrocarbons C_nH_n ($n = 14, 16, 18, 20, 24, 30$) has made available a class of compounds ('annulenes') of considerable

theoretical interest. In cooperation with Prof. Sondheimer, Dr R. Wolovsky, and Dr Y. Gaoni, we have undertaken structure analyses of two compounds in this series that are sufficiently stable to permit X-ray work. An account of the partial analysis of [14]annulene has been given by one of us (Bregman, 1962); preliminary results of the structure determination of [18]annulene were presented at the Cambridge meeting of the I.U.Cr. (Bregman & Rabinovich, 1960). The present paper reports the three-dimensional analysis of [18]annulene near 80 °K; the following paper (part II) gives the results of this study together with a discussion of the in-plane and out-of-plane molecular deformations.

Experimental

Crystals of [18]annulene, grown from slowly cooled solutions in chloroform, are thin monoclinic laths, dark brown, with occasional greenish sheen. They are elongated along [010], showing {001} and {20 $\bar{1}$ }, with pronounced cleavage along planes parallel to [010]. Room-temperature Cu $K\alpha$ photographs, which record few reflexions beyond $\sin \theta = 0.32$, yield the cell constants listed in Table 1. Systematically absent are reflexions $h0l$ with h odd, $0k0$ with k odd; the space group is $P2_1/a$. At the temperature of boiling nitrogen these crystals reflect throughout the reciprocal-lattice region accessible to Cu $K\alpha$ radiation; cell dimensions (Table 1) were determined from high-angle reflexions. The systematic absences of $P2_1/a$ were obeyed throughout the range of recorded reflexions. With two molecules of $C_{18}H_{18}$ per unit cell, the calculated density is 1.144 at 80 °K. The space group then demands that the molecule be centrosymmetric, unless the structure is disordered. This last contingency has been ruled out by refinement of an ordered model to a structure yielding acceptable agreement between F_o and F_c and showing no anomalies in the thermal parameters (see part II). [18] Annulene cannot, therefore, be formulated as a molecule with alternate single and double bonds.

Examination of a large number of crystals revealed the frequent occurrence of twinning across (001), made possible by the near equality (within 1.2%) of c^* and $4a^*\cos\beta^*$. The twinned crystals give, at room temperature, X-ray photographs that appear to have been produced by crystals having a doubled c axis and showing non-space-group extinctions for h even, $L (=2l)$ odd. At low temperature high-angle spots are found to be split, suggesting that the phenomenon observed is one of twinning rather than of polymorphism or

super-lattice formation. This conclusion is confirmed by the fact that the positions and intensities of all spots appearing on photographs of a twinned crystal can be exactly simulated by the superposition in mirror orientation of pairs of reciprocal-lattice nets derived from photographs of untwinned crystals. This superposition brings into near coincidence spots due to reflexions h, k, l and $h, k, -\frac{1}{2}(h+2l)$.

Reflexions from an untwinned crystal were recorded with Cu $K\alpha$ radiation by the equi-inclination Weissenberg technique in levels perpendicular to [010] up to $k=3$. The crystal was held near the temperature of boiling nitrogen by the device described by Hirshfeld & Schmidt (1956). From a crystal trimmed with a razor blade $0kl$ intensity data were collected that enabled us to place $I(hkl)$ ($k=0, 1, 2, 3$) on a common scale. However, the quality of cut crystals was inadequate to permit intensity photographs about another axis. The numbers of reflexions, observed plus unobserved, were 183, 356, 335, and 284, respectively, for $k=0, 1, 2$, and 3. Ten $04l$ reflexions were also included in the refinement process.

The intensities were estimated visually and corrected in the usual way; corrections for spot-shape extension, due to Phillips (1956), were applied to the data from upper-level photographs.

Solution and refinement

Comparison of the cell dimensions of [18]annulene and of coronene (Table 1) suggests a similarity in molecular shape and packing arrangement; since the strong high-order $h0l$ reflexions (207; 405; 602; 10,0,3; 14,0,1; 14,0,8; 16,0,1), used by Robertson & White (1945) for the solution of the latter structure, were, with the exception of 14,0,1, outstandingly strong in [18]annulene too, the x, z parameters of the peripheral carbon atoms of coronene were inserted into $F(h0l)$ structure-factor and least-squares calculations. The correctness of this model quickly became evident.

Refinement was accomplished on WEIZAC by a least-squares program modelled after the procedure described by Rossmann, Jacobson, Hirshfeld & Lipscomb (1959). This program seeks to minimize the discrepancy function

$$r = \sum_w (K^2 F_o^2 - F_c^2) / \sum_w K^4 F_o^4 \quad (1)$$

with respect to the scale factor K^2 and the positional and thermal parameters of the atoms in the asymmetric unit. It uses the linear diagonal approximation for

Table 1. Crystal data for [18]annulene and coronene

	a (Å)	b (Å)	c (Å)	β	Z	Space group
$C_{18}H_{18}$ at 80 °K	14.889 ± 0.004	4.800 ± 0.002	10.235 ± 0.003	111.60° $\pm 0.14^\circ$	2	$P2_1/a$
$C_{18}H_{18}$, room temperature	15.33	4.88	10.27	111.8°	2	$P2_1/a$
Coronene, room temperature	16.10	4.695	10.15	110.8°	2	$P2_1/a$

the computation of the shifts in all parameters, except that the scale factor K^2 and an average, isotropic, thermal parameter are adjusted separately by solution of an appropriate 2×2 matrix (Cruickshank, 1961).

For each reflexion, the weighting factor w_{hkl} was obtained by the summation of weights assigned to the individual spots on the several films on which the reflexion was recorded. These individual weights were a function of the film blackening and of the factors required for translation of the spot readings into scaled values of F_o^2 . This procedure yielded weights that varied roughly as F_o^{-4} for most reflexions, with decreased weights for the weak and the very strong reflexions. The summations in equation (1) comprise all except those unobserved reflexions for which $|F_c| < KF_t$, where F_t is a threshold value estimated for each unobserved reflexion. Scattering-factor curves were taken from Berghuis, Haanappel, Potters, Loopstra, MacGillavry & Veenendaal (1955) for carbon and from McWeeny (1951) for hydrogen.

Refinement of the (010) projection proceeded satisfactorily, initially with isotropic carbon atoms only. Next, hydrogen coordinates derived from a difference synthesis $\Delta\rho(x, z)$ were inserted into the least-squares

calculations, and the refinement of the carbon parameters was continued for several additional cycles.

In the next stage y parameters of all atoms were derived for a hypothetical planar molecule; this model was tested by structure-factor calculations for $F(h1l)$ and found to give reasonable agreement.

Because of the diagonal approximation used in the least-squares program, it was desirable to refer the structure to nearly orthogonal axes. The coincidence in cell dimensions responsible for the twinning (see above) provided such a coordinate system, based on a unit cell four times the size of the primitive cell. The axes of this cell, having a monoclinic angle of 90.26° , are obtained from the conventional axes by the transformation whose matrix is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

In the first four three-dimensional refinement cycles, only the scale factor K^2 , the y parameters, and the isotropic thermal parameters (B) of the carbon atoms were varied; next all the carbon parameters were re-

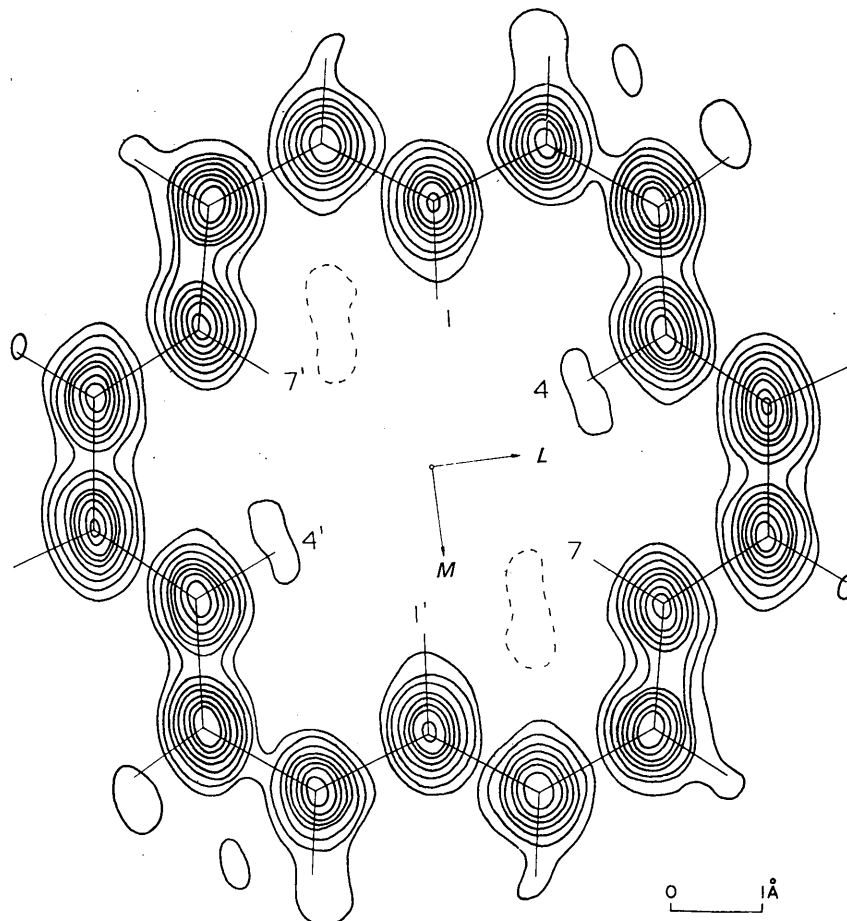


Fig. 1. Electron density in mean molecular plane. Contours at intervals of $1 \text{ e.}\text{\AA}^{-3}$, zero contour broken. Molecular outline shows projected atomic positions based on least-squares refinement.

Table 2. *Atomic coordinates*

Atom	Referred to crystal axes			Referred to molecular axes		
	<i>x</i>	<i>y</i>	<i>z</i>	<i>L</i> (Å)	<i>M</i> (Å)	<i>N</i> (Å)
C(1)	-0.1147	-0.4574	0.0430	0.424	-2.881	0.061
C(2)	-0.1068	-0.5089	0.1807	1.742	-3.323	-0.015
C(3)	-0.0441	-0.3627	0.2988	2.864	-2.467	-0.058
C(4)	0.0151	-0.1483	0.2903	2.775	-1.091	-0.053
C(5)	0.0836	-0.0082	0.3995	3.807	-0.178	0.010
C(6)	0.1430	0.2077	0.3781	3.594	1.236	0.041
C(7)	0.1416	0.2877	0.2474	2.343	1.828	0.085
C(8)	0.1885	0.5084	0.2154	2.034	3.162	-0.013
C(9)	0.1771	0.5884	0.0770	0.715	3.675	-0.064
H(1)	-0.072	-0.278	0.031	0.31	-1.79	0.00
H(2)	-0.143	-0.668	0.197	1.90	-4.30	0.04
H(3)	-0.043	-0.427	0.390	3.74	-2.91	-0.06
H(4)	0.006	-0.078	0.195	1.86	-0.69	-0.14
H(5)	0.095	-0.054	0.499	4.76	-0.47	0.05
H(6)	0.187	0.320	0.464	4.41	1.87	-0.02
H(7)	0.107	0.168	0.174	1.64	1.24	0.25
H(8)	0.227	0.626	0.297	2.81	3.77	-0.15
H(9)	0.211	0.749	0.060	0.55	4.63	-0.16

fined. After several further cycles anisotropic carbon thermal parameters β_{ij} ($i, j=1, 2, 3$) were introduced (Cruickshank, 1956), and only when the nine parameters per carbon atom had reached essentially stationary values were the positional and isotropic thermal parameters of the hydrogen atoms allowed to vary. After 28 cycles the discrepancy factor r had reached 0.0372.

At this stage bond lengths and angles were calculated, as well as the equation of the best plane through the carbon skeleton and the deviations of the carbon and hydrogen atoms from this plane. The carbon skeleton conformed closely to symmetry $\bar{3}$; on the other hand, the displacements of hydrogen atoms from the mean plane did not follow a similar pattern. In particular, the position of H(1) appeared, on the criterion of molecular symmetry $\bar{3}$, to be in serious error. A three-dimensional difference synthesis based on $F_o - F'_c$ (with the sign of F'_c , including hydrogen contributions, attached to F_o ; carbon contributions alone for F'_c) was computed in the neighborhood of H(1) and H(4). While for the latter the electron-density maximum coincided within 0.02 Å with the position derived by the least-squares procedure, the electron density near H(1) indicated a position 0.11 Å above, rather than 0.14 Å below, the mean plane of the carbon atoms. This more plausible location for H(1) was inserted into further least-squares calculations, but these returned the atom to its former position.

The entire set of intensity data was re-checked for human errors, and minor corrections were made in the intensities of about 100 reflexions, especially at high angles. About 60 further reflexions were assigned zero weights because of high background scattering or because of suspected interference by the beam catcher. New hydrogen coordinates were postulated, based on the last set of carbon coordinates and the assumption that each C-H bond, of length 1.00 Å, lay in the plane defined by the three adjacent carbon atoms and bisected the C-C-C angle. The positional and thermal parameters of the carbon atoms were refined for another five cycles, after which the hydrogen parameters were again allowed to change.

The final hydrogen positions did not show any significant improvement, though the anomaly in H(1) was smaller than before. A new three-dimensional difference synthesis, calculated in the same way as the previous one, showed maxima at positions differing by up to 0.14 Å from the hydrogen positions given by least-squares. All hydrogen atoms showed marked anisotropy; the electron-density distributions resembled

Table 3. *Direction cosines of the molecular inertial axes with respect to the crystal axes*

	<i>a</i>	<i>b</i>	<i>c</i>
<i>L</i>	-0.0393	0.0266	0.9433
<i>M</i>	0.6593	0.7517	-0.2370
<i>N</i>	0.7509	-0.6587	-0.2325

Table 4. *Estimated standard deviations (average) of atomic coordinates and thermal tensor components, referred to the edges of a nearly orthogonal cell having $a=14.889$, $b=4.800$, $c'=38.059$ Å, $\beta'=90.26^\circ$*

(This is the cell used for the diagonal least-squares refinement)

Carbon:

$$\begin{array}{lll} a\sigma(x') = 0.0024 \text{ \AA} & b\sigma(y) = 0.004 \text{ \AA} & c'\sigma(z') = 0.0023 \text{ \AA} \\ \sigma(U'_{11}) = 0.0010 \text{ \AA}^2 & \sigma(U'_{22}) = 0.0026 \text{ \AA}^2 & \sigma(U'_{33}) = 0.0010 \text{ \AA}^2 \\ \sigma(U'_{12}) = 0.0013 \text{ \AA}^2 & \sigma(U'_{23}) = 0.0014 \text{ \AA}^2 & \sigma(U'_{13}) = 0.0008 \text{ \AA}^2 \end{array}$$

Hydrogen:

$$\begin{array}{lll} a\sigma(x') = 0.030 \text{ \AA} & b\sigma(y) = 0.038 \text{ \AA} & c'\sigma(z') = 0.027 \text{ \AA} \\ & \sigma(u^2) = 0.008 \text{ \AA}^2 & \end{array}$$

Table 5 (cont.)

<i>h</i>	<i>k</i>	<i>l</i>	100 <i>F</i> _o	100 <i>F</i> _c	<i>h</i>	<i>k</i>	<i>l</i>	100 <i>F</i> _o	100 <i>F</i> _c
-11			302	281	-14			619	-995
-10			<251	11	-13			<279	-179
-9			945	897	-12			295	314
-8			243	131	-11			1790	1876
-7			243	-212	-10			1393	1386
-6			236	309	-9			1540	1524
-5			670	-682	-8			369	-354
-4			999	-983	-7			590	-538
-3			1274	1346	-6			646	-604
-2			228	-205	-5			363	295
-1			<198	86	-4			228	-182
0			988	896	-3			700	652
1			592	-592	-2			<163	-21
2			<243	55	-1			626	620
3			574	564	0			257	-225
4			703	-752	1			177	7
5			150	515	2			862	-934
6			179	179	3			485	-501
7			221	221	4			<214	55
8			243	243	5			988	973
9			<243	-283	6			1000	952
10			<243	186	7			243	-236
11			452	519	8			<251	-215
12			177	-105	9			677	-646
13			74	-310	10			<243	-182
14			228	-755	11			610	654
15			228	-187	12			<244	225
16			1237	-105	13			<205	93
17			<251	-1273	14			353	-415
18			603	-1273	15			236	314
19			603	-649	16			426	419
20			808	-1	17			244	110
21			351	792	18			116	430
22			1384	227	19			442	-300
23			358	-1309	20			677	-691
24			1479	-425	21			<251	77
25			413	-1605	22			893	-809
26			<205	434	23			334	-317
27			1008	-92	24			1119	-1025
28			221	1050	25			<205	12
29			279	-329	26			456	409
30			403	-259	27			<177	14
31			275	932	28			603	845
32			552	-549	29			221	486
33			<251	570	30			1145	-954
34			512	345	31			2003	2056
35			<205	89	32			<163	-94
36			<252	8	33			302	-341
37			179	179	34			610	-779
38			670	179	35			1347	1293
39			153	-710	36			1355	1396
40			153	-146	37			2183	2247
41			153	-249	38			<243	293
42			641	-105	39			<251	-11
43			311	-311	40			1203	-1048
44			<205	104	41			<236	-146
45			641	637	42			221	151
46			856	856	43			313	351
47			654	446	44			<189	147
48			946	-1053	45			<163	80
49			190	-649	46			171	-100
50			492	-174	47			382	323
51			177	177	48			684	678
52			673	673	49			<228	-185
53			574	-574	50			390	-375
54			304	304	51			1020	-1169
55			782	688	52			647	795
56			153	364	53			927	-846
57			<205	-153	54			302	-254
58			673	-362	55			398	-286
59			153	-562	56			1355	-1251
60			490	-776	57			310	759
61			345	-342	58			1221	1187
62			153	153	59			<145	-14
63			153	153	60			348	470
64			153	153	61			591	-679
65			153	153	62			228	-183
66			153	153	63			1379	1735
67			810	79	64			<243	-178
68			500	214	65			642	-667
69			214	-235	66			<276	8
70					67			<276	20
71					68			571	639

prolate spheroids with their long axes nearly normal to the molecular plane. Final values of r and $R = \Sigma |KF_o - |F_c|| / \Sigma KF_o$ were 0.0289 and 0.076, respectively; unobserved reflexions for which $KF_i < |F_c|$ were included in the sums.

Observed and calculated structure factors are compared in Table 5.

An electron-density section in the mean molecular plane is shown in Fig. 1.

The final coordinates of all atoms in the asymmetric unit, derived from the last cycle of least-squares refinement, are listed in Table 2 with respect both to the crystal axes and to a set of orthogonal molecular axes. The latter (Table 3) were chosen to coincide with the principal molecular moments of inertia, calculated for the carbon atoms only, by the same computer program that yielded the equation of the mean plane according

to the method of Schomaker, Waser, Marsh & Bergman (1959). Because the molecule has very nearly $\bar{3}$ symmetry (see part II) the moment of inertia is almost axially symmetric; thus the directions of the in-plane inertial axes L and M have slight physical significance.

Table 4 gives the average standard deviations computed by the diagonal approximation:

$$\sigma^2(u_i) = \Sigma w(K^2 F_o^2 - F_c^2)^2 / (n-s) A_{ii} \quad (2)$$

where $n=911$ is the number of reflexions, 883 observed plus 28 threshold, included in the summations of the last refinement cycle, $s=118$ is the number of parameters u_i adjusted, and $A_{ii} = \Sigma w(\partial F_c^2 / \partial u_i)^2$ is the diagonal element in the normal equation corresponding to the parameter u_i . The approximate validity of the diagonal approximation follows from the near orthogonality of the axes and the large number of reflexions observed.

Fig. 2 shows the final values of bond lengths and angles and the distances of all atoms from the mean plane of the carbon atoms. This plane has the equation

$$11.1795x - 3.1616y - 2.3798z = 0,$$

in which the left side gives the distance from the plane, in Å, of a point having fractional coordinates (x, y, z).

The results of this analysis are discussed in part II.

We are grateful to Professor Franz Sondheimer and Dr Reuven Wolovsky, who brought this problem to our attention and supplied the crystals for this investigation. It is a privilege to acknowledge our indebtedness to the late Professor I. Fankuchen, whose suggestion led to the proper diagnosis of the twinning phenomenon.

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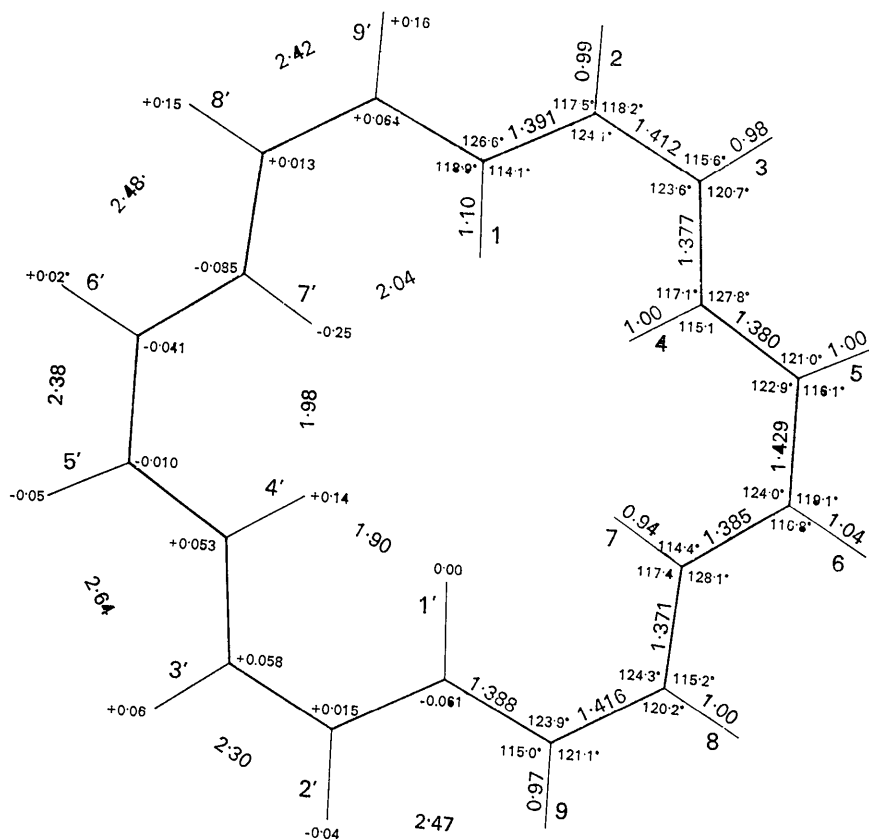


Fig. 2. Interatomic distances (Å) and bond angles. Figures near atomic positions give displacements (Å) from mean molecular plane.

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